A Low-Noise X-Band Parametric Amplifier Using a Silicon Mesa Diode*

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Summary—**This paper summarizes a cooperative effort to develop silicon mesa variable-capacitance diodes and to evaluate their potential for achieving low-noise amplification in the high microwave frequency range. Cutoff frequencies of about 70 kMc at zero-bias voltage (corresponding to 140 kMc at maximum reverse bias voltage) with a total permissible voltage swing in excess of 5 volts have been obtained.**

A versatile degenerate X-band parametric amplifier was developed which, when used in conjunction with these silicon mesa diodes, achieved a radiometer noise temperature of 130°K at 8.5 kMc with a 50-Mc bandwidth at 17-db gain. The measured performance of the diode (figure of merit) is compared with the first-order theory in an operating radar system. The over-all performance of the amplifier improved the observed system sensitivity by 6 db.

IN this section, we present a first-order design theory which suggests that practical silicon-mesa-diode cutoff frequencies measured at zero bias have a theoretical upper limit well in excess of 200 kMc. In the laboratory, approximately 30 per cent of this theoretical value has been achieved; however, it is important to note that the measured value of the cutoff frequency has been remarkably substantiated by the measurement of noise temperature at X band.

The objective is to derive an expression for the cutoff frequency at zero bias from the definition

$$f_c(0) = \frac{1}{2\pi R_s C(0)} \tag{1}$$

in terms of pertinent parameters: parent material resistivity ρ, diameter of the mesa d, and the diffusion length in forming the junction L. The total series resistance of the mesa is approximately the sum of the axial length of the mesa l and the constriction resistance at the base of the mesa. In terms of volume resistivity ρ, this is given by

$$R_s \simeq \frac{\rho}{d}\left(\frac{1}{2} + 1.27\frac{l}{d}\right) \text{ ohms.} \tag{2}$$

Since in practice the l/d ratio is of the order unity, (2) is approximately given by

$$R_s \simeq 0.70\frac{\rho}{d} \text{ ohms,} \tag{3}$$

where ρ is in units of 10^{-3} ohm-cm and d is in units of 10^{-3} inches. The capacitance per unit area of a linearly graded junction in silicon is found from Shockley's expression[1] in terms of practical units:

$$C(0) = 2.53 \times 10^{-3} a^{1/3} V_0^{1/3}\ \mu\mu\text{f}, \tag{4}$$

where V_0 is the total voltage across the junction and the concentration gradient a at the junction is given by

$$a = \frac{N_s}{\sqrt{\pi} L}\exp\left(-\frac{x_j^2}{4L^2}\right), \tag{5}$$

where N_s is the constant surface concentration, L is the diffusion length, and x_j is the junction depth. Substituting (5) into (4) yields for the capacitance

$$C(0) = 1.75 d^2 L^{-1/3}\ \mu\mu\text{f} \tag{6}$$

where

$$N_s = 4 \times 10^{20}\ \text{cm}^{-3}$$
$$V_0 = 0.6 \text{ volt}$$

and

L is in units of microns

has been assumed for the silicon parent material. We may now now express the zero-bias cutoff frequency (1) in terms of the design parameters:

$$f_c \simeq 131\frac{L^{1/3}}{\rho d}\text{ kMc} \tag{7}$$

Inspection of (7) shows that the cutoff frequency is increased by starting with the lowest resistivity silicon feasible from the standpoint of crystal growth and yet consistent with the ability to overcompensate the parent material by diffusion to form a p-n junction. The mesa structure should be reduced to the minimum diameter to achieve reasonable admittance levels for high microwave frequency applications, but it cannot be reduced without limit because of practical considerations, some of the more important of which are strength and contact resistance. Because of the cube-root dependence, the diffusion length is not critical. Since, in practice, an etched mesa has a conical shape, deeper diffusion causes a greater increase in capacitance than can be offset by an

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[1] W. Shockley, "The theory of p-n junctions in semiconductors and p-n junction transistors," *Bell Syst. Tech. J.*, vol. 28, pp. 435–489; July, 1949.

increase in diffusion length. For these reasons, we have found it practical to form a mesa of 8×10^{-4} inches in diameter in 10^{-3} ohm-cm silicon parent material with a diffusion length of about 3 microns. This resistivity and diffusion length are also in keeping with the minimum reverse breakdown voltage, as can be seen from Fig. 1. Eq. (7), which incidentally shows a dependence of the cutoff frequency on the mesa diameter not heretofore appreciated, is plotted for these parameters as a function of the concentration gradient, or diffusion length. Although this curve shows continued improvement in cutoff frequency at high diffusion lengths, the above practical considerations tend to establish a maximum value for this type of geometry not too far from the 3-micron value, which is the diffusion length of our choice. Four measured points are shown in Fig. 1. The dot represents a low-frequency estimate of the cutoff frequency at zero bias which we obtained by using the capacitance measured at 100 kc and the resistance corresponding to the slope of the I-V diode characteristic at large forward currents. The crosses denote measured cutoff frequencies at 9 kMc for three different diodes. In practice, a cutoff frequency of about 30 per cent of the predicted value or about 70 kMc (as measured by standard techniques described in the recent literature) has been obtained. It is not at all surprising that the obtained cutoff frequency is considerably less than the predicted one, when it is recognized that the calculations of the cutoff frequency apply strictly to a low-frequency model and

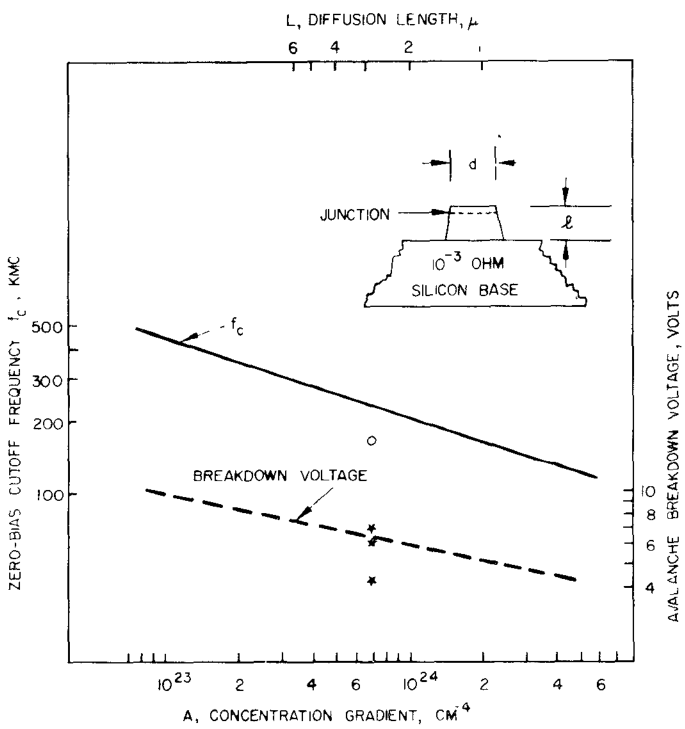

Fig. 1—Cutoff frequency of silicon mesa diode as a function of diffusion length L (microns) and concentration gradient a (cm^{-4}) at the junction for $\rho=10^{-3}$ ohm-cm, $N_0=4\times10^{20}$ cm^{-3}, and $V_0=0.6$ volt.

do not account for losses which may be limiting factors at very high frequencies. These factors are presently under investigation. It is for this reason that accurate prediction of noise performance demands that the cutoff frequency be measured in the vicinity of the expected operating range. The change between the dc measurement and one conducted at 9.0 kMc is indicated in the figure, and it is this latter value that must be used to predict performance.

Figure of Merit and Predicted Performance

The cutoff frequency at zero bias and maximum voltage swing without the current flow in either direction through the diode are a measure of the figure of merit. This figure of merit is valid in predicting the noise performance of a parametric amplifier, if it can be shown that the noise sources are entirely resistive. The figure of merit which has been suggested[2,3] and previously used to predict the performance of parametric amplifiers at various microwave and UHF frequencies is given by

$$K = \left(\frac{\Delta C}{2\bar{C}}\right)\left(\frac{f_c}{f_s}\right), \tag{8}$$

where

ΔC is the capacitance swing at maximum voltage swing,
$\bar{C}$ is the mean capacitance,
f_s is the operating frequency,
f_c is the cutoff frequency at optimum bias voltage.

To evaluate the quality of a variable-capacitance diode in terms of this figure of merit, we have built a parametric amplifier at X band in which both signal and idler frequencies are supported in the same cavity. As will be described later, this type of amplifier can be made very flexible and is uniquely suited for this measurement. This figure of merit enters into performance as follows: The radiometer noise temperature[4] of such an amplifier is given by

$$T_{n_2} = \frac{1}{\mu_{s_2}} T_d, \tag{9}$$

where $\mu_{s_2} \leq K-1$, T_d is the diode body temperature, and the equality holds for maximum available capaci-

[2] R. C. Knechtli and R. D. Weglein, "A Lower Limit on the Noise Temperature of a Parametric Amplifier," presented at the 17th Conf. on Electron Tube Res., Mexico D. F., Mexico, June 24, 1959.

[3] R. C. Knechtli and R. D. Weglein, "Low noise parametric amplifier," Proc. IRE, vol. 48, pp. 1218–1226; July, 1960.

[4] Two kinds of noise temperatures are commonly referred to in connection with parametric amplifiers. When signal and idler frequencies are relatively near each other, as in this example, both frequency ports are approximately equally loaded by the source conductance, and we speak of a radiometer noise temperature, indicating that all available frequency ports are connected to the source. The alternative method of reception, where the signal source is connected to one frequency port, though others may receive and generate noise, is often characterized by the term, "radar noise temperature."

tance swing. The pump power required to maintain high gain and this noise temperature is given by

$$\frac{P_2}{(\omega_s\bar{C})\left(\frac{\Delta C}{2\bar{C}}\right)(\Delta V)^2} \geq \frac{1}{2K}, \tag{10}$$

where the equality applies to optimum conditions of matching and maximum utilization of the available capacitance swing. The pump power has been normalized to the design parameters, *viz.*, admittance level, normalized capacitance, and the maximum voltage swing.

The capacitance-voltage characteristic of a typical silicon mesa diode as measured at 100 kc is shown in Fig. 2; the package parasitic capacitance is also included. In Fig. 3, the dc characteristic of this diode shows the maximum voltage swing for less than 1 μa current in either direction to be 5.5 volts. The normalized capacitance swing $\Delta C/2\bar{C}$ corresponding to this voltage swing was computed from Fig. 2,[5] and the cutoff frequency was measured at 9 kMc. The resulting figure of merit and the derived predicted performance are shown in Table I. These numbers represent the best anticipated behavior under the previously stated optimum conditions. The diode body temperature has been assumed to be the ambient (300°K) temperature.

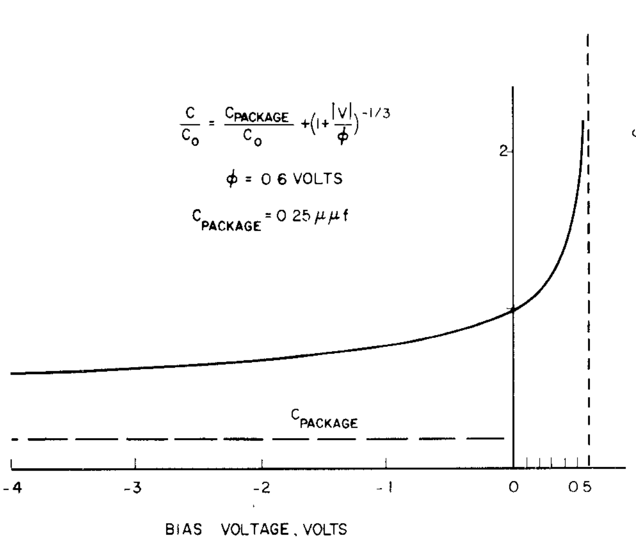

Fig. 2—Capacitance-voltage curve of X-band silicon mesa diode as measured at 100 kc. $C_{package}=0.25$ pf; $\phi=0.6$ volt is the built-in voltage.

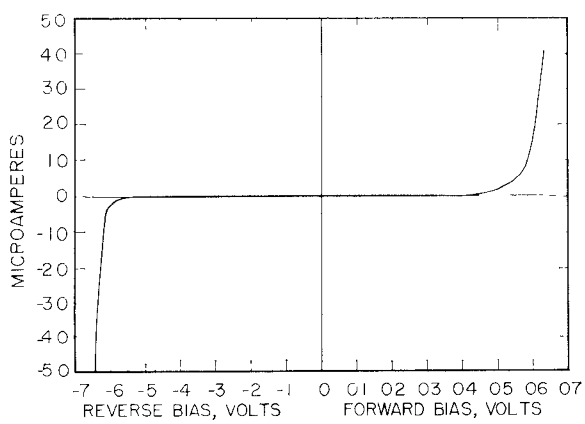

Fig. 3—Measured dc characteristic of silicon mesa diode used in experiment.

TABLE I

DC bias voltage	2.5v
$\Delta C/2C$	0.26
f_c (0 bias)	68 kMc
f_c (operating bias)	108 kMc
Signal frequency	8.5 kMc
Figure of merit	3.1
Noise temperature	140° K
Pump power	75 Mw

Philosophy of Amplifier Circuit Design

There is little doubt that in order to obtain low-noise broad-band performance from a parametric amplifier at a given frequency, not only is it necessary to use a diode with the best available Q, but also the circuit in which the diode is to be used must offer those complementary features which permit the achievement of the optimum performance. Thus, either the circuit design must take the exact parameters of the diode into account or, if these are not accurately known, sufficient flexibility must be built in to cope with the expected range of the variables. This last feature of flexibility is of particular importance in an amplifier circuit which is to perform at X band. Here, the diode has a self-resonant frequency, most likely below the signal frequency, and measures must be taken to raise this resonance to a more favorable place, if possible above the idler frequency. If this is not done and the cavity is relied upon to tune out the inductance at the signal frequency, not only is the noise performance likely to be impaired because of the added resonance losses but also the bandwidth will surely be curtailed. An additional consideration is the ratio of the diode mean capacitive susceptance to the cavity characteristic admittance. It can be shown that the susceptance variation through the amplification band is minimized when this ratio is of the order of unity. In many cases, it is not practical to fulfill this requirement. For example, in this amplifier the lowest practical characteristic impedance of the X-band cavity was about 100 ohms, while the diode reactance at the operative bias was approximately 30 ohms.

[5] S. Sensiper and R. Weglein, "Capacitance and charge coefficients for parametric diode devices," Proc. IRE, vol. 48, p. 1482; August, 1960.

The size of the signal and idler cavities is a final consideration in achieving a desired bandwidth. For maximum bandwidth, they should be as small as possible and should resonate in the fundamental mode to keep the reactance variation across the band at a minimum. To achieve the desired flexibility, we designed the diode holder to permit an arbitrary diode position in the signal and pump cavities and incorporated a coaxial tuner as an extra degree of freedom in order to manage the diode self-resonance. The actual circuit configuration which resulted from the incorporation of these features is shown in Fig. 4. While little mention has been made of circuit properties which yield a minimum noise performance, the above features, which ensure flexibility and bandwidth, also tend to ensure low-noise performance if the circuit losses are kept at a minimum. The built-in flexibility then permits sufficient overcoupling of the source or antenna conductance to minimize the contributions from diode losses without seriously restricting the bandwidth criteria.

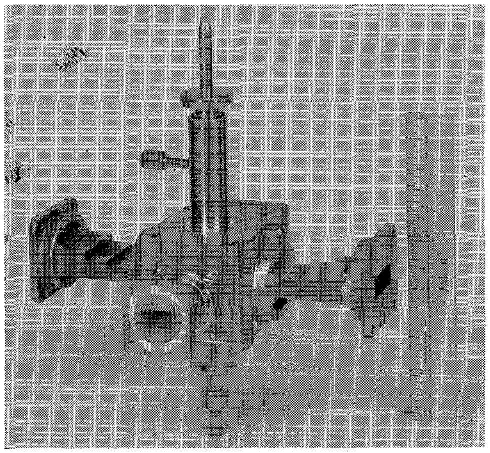

Fig. 4—Degenerate *X*-band amplifier showing transformers to standard waveguide sizes.

Results

The performance of this amplifier with a number of these diodes was measured by use of the arrangement illustrated in Fig. 5. Provisions were made for observing both bandwidth and noise performance at separate switch positions. Because of the pitfalls in measuring noise figures of negative resistance amplifiers, considerable precautions were taken to make the noise measurements as accurate as possible. All losses were carefully determined, and the noise figure of the balanced mixer following the parametric amplifier was measured. The quoted noise temperatures of the parametric amplifier were then referred to its input terminals. In Table II, the measured performance is compared with the predicted one; the numbers with an asterisk are the measured values in each case. The best performance was obtained with diode No. 2, which gave a radiometer noise temperature of 130°K and a gain-bandwidth product of

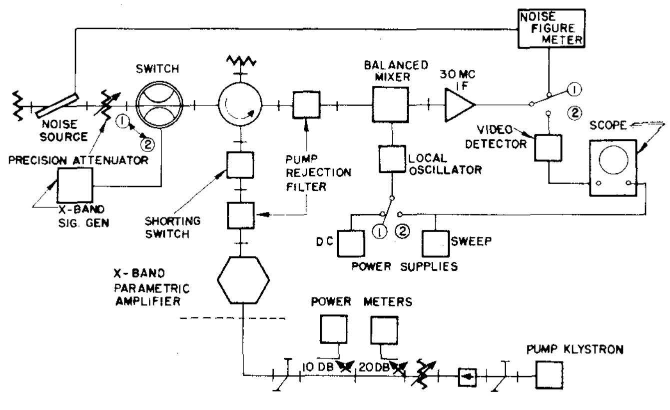

Fig. 5—Schematic of circuit arrangement to measure gain-bandwidth (position ②) and noise performance (position ① shown).

TABLE II

Diode No.	1		2		3	
Cutoff frequency f_c kMc, 0-bias voltage	62		68		44	
Figure of merit K	2.8		3.1		1.2	
Noise temperature, °K	170	240*	140	130*	250	260*
Pump Power, Mw	80	150*	74	145*	100	180*
Gain, db	17		17		19	
Bandwidth, Mc	56		50		10	

350 Mc, *i.e.*, a bandwidth of 50 Mc at 17-db insertion gain.

The performance of the amplifier was checked for compatibility with existing radar systems. Before the parametric amplifier was included, the system noise figure was measured and found to be about 10 db. From the measured performance of the parametric amplifier, the increase in sensitivity of the over-all system was estimated to be between 5 db and 6 db. Fig. 6 shows an A-scope display, from which the actual improvement in sensitivity can be estimated. This presentation shows amplitude of the reflected signal vs range; the large amplitude at the left is the transmitted pulse. The top portion of the figure represents the receiver sensitivity prior to the inclusion of the parametric amplifier. For the same background noise level, it is seen that in the bottom picture, where the low-noise parametric amplifier increases the sensitivity, the low-level signals appear to be decidedly stronger. A particularly good example is indicated by the arrow, from which an approximate increase in sensitivity commensurate with the prediction can be seen.

After the initial evaluation of these silicon mesa diodes was completed and their potential in low-noise parametric amplifiers had been well established, a compact amplifier was designed. It was specifically made compatible with these particular diodes in order to bring out their optimum features with a minimum of

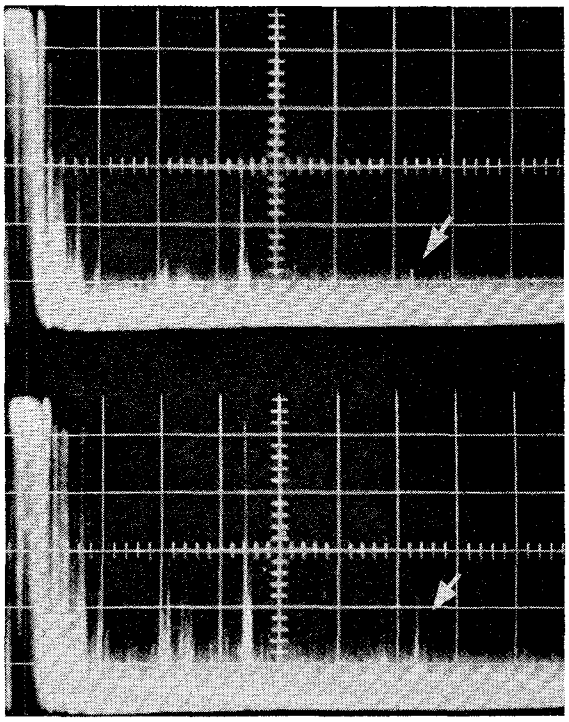

Fig. 6—Radar presentation showing improvement of sensitivity through inclusion of parametric amplifier.

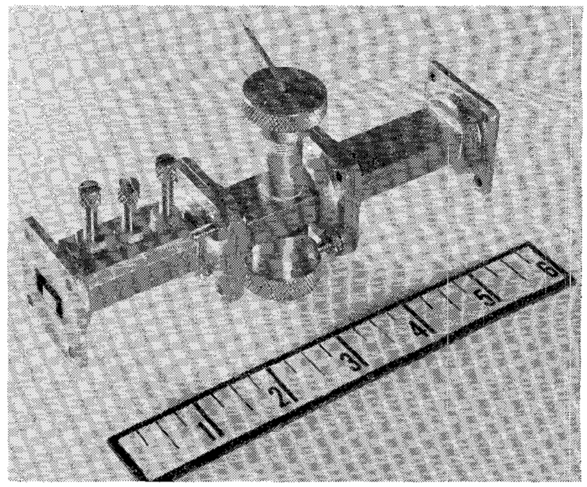

Fig. 7—Developmental model of degenerate X-band parametric amplifier specifically adapted to silicon mesa diode.

TABLE III

Frequency	8.5 kMc
Power gain	15 db
Bandwidth	20 Mc
Radiometer Noise figure	1.0 db (80°K)
Pump power	45 Mw

adjustments. Thus, the flexibility of the previously described experimental amplifier was sacrificed. An early developmental model is seen in Fig. 7. Both the noise performance and the pump power requirement improved, as Table III reveals. This enhancement can be directly related to the fairly precise knowledge of the particular diode characteristics which were used in arriving at the compatible circuit design. Although the gain-bandwidth product in this model was somewhat less than expected, further modifications of this design are expected to yield considerable improvements.

Conclusions

The design of silicon mesa variable-capacitance diodes has successfully led to devices with cutoff frequencies and figures of merit that make low-noise reception at X band a practical reality. Zero-bias cutoff frequencies of nearly 70 kMc led to a noise temperature of 130°K in an X-band parametric amplifier which had a gain-bandwidth product of 350 Mc. Compatibility with existing radar systems was demonstrated. Precise knowledge of the diode characteristics is mandatory to achieving the optimum performance. As a corollary, in order to maintain uniformly high performance in a particular circuit design, where many similar units are involved, such as in a production model or in traveling-wave parametric amplifiers, uniform diode characteristics (capacitance-voltage curve as well as cutoff frequency) are of the utmost importance.

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